Title: Constructing Sierpinski's Triangle

Brief Overview:

Students will review the basic techniques necessary to construct congruent line segments, midpoints, perpendicular bisectors, and congruent angles. The students will then be introduced to Sierpinski's triangle through the Chaos game described in the lesson. Afterwards, the student will CONSTRUCT and color their own Sierpinski's triangle.

Links to NCTM Standards:

- **Mathematics as Problem Solving**
  Students will use the problem solving method of skill acquisition and skill application.

- **Mathematics as Communication**
  Students will discuss, then write their interpretation(s) of the results of the Chaos game.

- **Mathematics as Reasoning**
  Students will be asked to suggest reasons the results of the Chaos game were such as what they were.

- **Mathematical Connections**
  Students will be asked to suggest reasons the Chaos game produced the repetitive triangular pattern.

- **Patterns and Functions**
  The patterns of fractals will be introduced on a general level through Sierpinski's triangle.

- **Probability**
  Probability discussions can be generated through the die roll and the effect it has on the placement of the dot in the Chaos game.

- **Geometry**
  Geometric principles will be developed through the construction of line segments, perpendicular bisector, congruent angles and the discussion of equilateral triangles.

- **Measurement**
  Students will use both a compass and a protractor as a measuring tool.

Grade/Level:

Grades 5 (gifted) - 10

Duration/Length:

This lesson can take a single day or as long as three days depending on the length of the class period, student ability, or depth and breadth of discussions generated.

Prerequisite Knowledge:

Students should have working knowledge of the following skills :

- Working with a compass, protractor, straightedge

- Basic angle and triangle classification
- General probability (as refers to roll of die)

Objectives:

Students will:

- construct line segments congruent to given segments, perpendicular bisectors, and angles congruent to given angles.
- learn to recognize a Sierpinski's triangle.
- construct a Sierpinski's triangle.
- be introduced to the basic concept of fractals and their repetitive nature.

Materials/Resources/Printed Materials:

- Compass, protractor, straightedge
- Pen, pencil, coloring tools
- Overhead transparencies, overhead pens (sets of three colors: red, blue, green)
- Several dice (enough to distribute one to each group of three or four students)
- Construction Assessment worksheet
- Chaos Game worksheet
- Sierpinski's Triangle Construction worksheet.

Development/Procedures:

(**NOTE:** the days allotted here are based on the average grade 6-8 class in a 40-45 minute period)

DAY 1

- Give each student a compass and a straightedge.
- Using a compass and a straightedge, demonstrate the constructions of a line segment, a segment bisector, and an angle congruent to a given angle. Methods for these constructions can be found in any good concepts of geometry text. Allow students ample time to practice each construction.
- Hand out **Construction Assessment Worksheet**. Depending on your schedule or student ability level this worksheet can be a homework assignment, a short quiz of skills attained during the lesson, or a classroom exercise. See the assessment rubric enclosed.

DAY 2

- Put students into groups of three or four.
- Give each group an overhead transparency and a set of overhead pens (three colors: red, blue, green).
- Give each student a copy of the Chaos Game worksheet.
- Allow enough time for the groups to run at least 20-25 repetitions. (The more repetitions the better the overall class results will be.)
- Have a representative of each group bring their transparency to the overhead and align the three vertices.
- Ask students to suggest reasons the points form this pattern. Steer the discussion towards the specific instructions for each roll of the die and the role it may have played in the placement of the dot. Also try to have them see the relationship between the pattern emerging and the construction of the bisectors from the earlier lesson.
- Introduce Sierpinski's Triangle as the name of the pattern that emerged from the Chaos Game. (There are many good books on fractals and chaos that can be used as references or motivational material for the next day's lesson. A good picture of Sierpinski's triangle can be found in just about any of them or a more recent geometry text.)

DAY 3

- Students will need a compass, straightedge, protractor, and coloring tools.
- Provide each student with a copy of the Sierpinski's Triangle Construction worksheet.
- Students are to construct an angle congruent to the angle (60°) given at the top of the worksheet at each end of the line segment provide at the bottom of the worksheet.
- Students then are to construct a line segment congruent to the given line segment (at bottom of worksheet) along the terminal ray of each angle previously constructed. This will form an equilateral triangle. Have students verify that their triangle is equilateral.
- Using the construction technique of the perpendicular bisector, students should find the **midpoint** of each side of their triangles. **Emphasize to the students that they should not actually draw the bisector but merely mark the spot on each side where its bisector would intersect that side.**
- Students will then connect each of these midpoints **with a straightedge**, creating three new upright triangles.
- Repeat this process with each successive upright triangle until the length of the sides of the resulting triangles is 1".
- Have students present their finished construction for approval.
- Students are to color their completed **and approved** constructions.

Performance Assessment:

General scoring rubric should be as follows:

4 - Good construction techniques with use of straightedge. Accurate representation of the segment or angle.
3 - Obvious construction technique, but less than accurate representation OR poor use of tools (straightedge/compass)
2 - Attempt at construction obvious, but poor representation AND poor use of tools
1 - Obvious faked construction, poor representations, little on no apparent use of tools.

Extension/Follow Up:

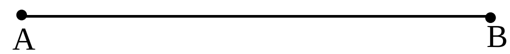
1. Have students develop equations by overlaying a Cartesian Coordinate plane with one vertex at the origin and the side lying along an axis. Then locate midpoints using the midpoint formula.
2. Examine implied equilateral triangles in nature. Have students look for examples in photos, magazines, etc. Also discuss the apparent Chaos of nature (river's paths, leaf patterns, coastlines) that appear to reiterate themselves. Have students examine the coastline of the U.S. on a good map with closer and closer detail. Does the "chaotic" nature of the coast have a repetitive pattern?
3 After each iteration, count the total number of triangles. Record iteration # and number of triangles as ordered pairs. Is there a functional relationship. How about well known number patterns (e.g., Fibonacci, Pascal triangle?)
4. Try subdividing the side by trisection, or into quarters, and then connecting the points. Is there a pattern? Can this be continued indefinitely like Sierpinski's?

Authors:

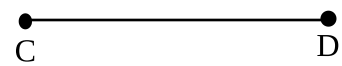
Kenny Garvey	Claire Fergurson
Governor Thomas Johnson High	CMST
Frederick County, Maryland	Univ. of MD. - Eastern Shore

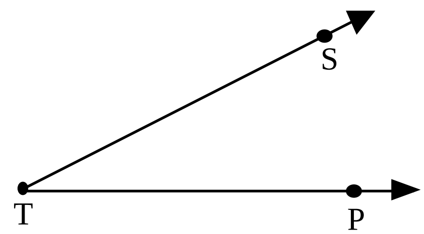
Construction Assessment Worksheet

1. Construct a line segment congruent to $\overline{AB}$

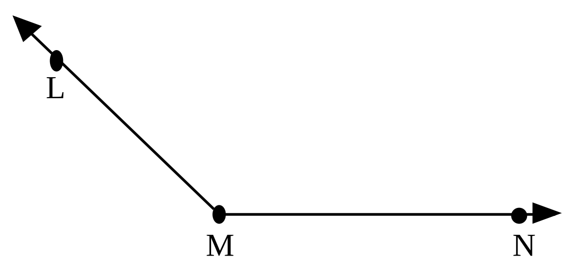


2. Construct a perpendicular bisector of line segment $\overline{CD}$



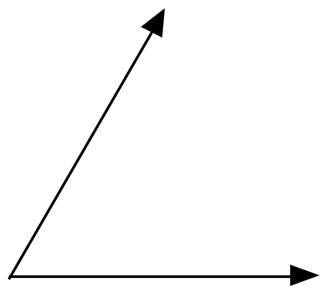


3. Construct an angle congruent to $\angle$STP



4. Construct an angle congruent to $\angle$LMN

Sierpinski's Triangle Worksheet



1. Construct two angles congruent to the angle at left on each end of the line segment at the bottom of this page.

2. Construct two segments congruent to the given segment along the terminal ray of your newly constructed angles. (These angles and segments should complete an equilateral triangle.)

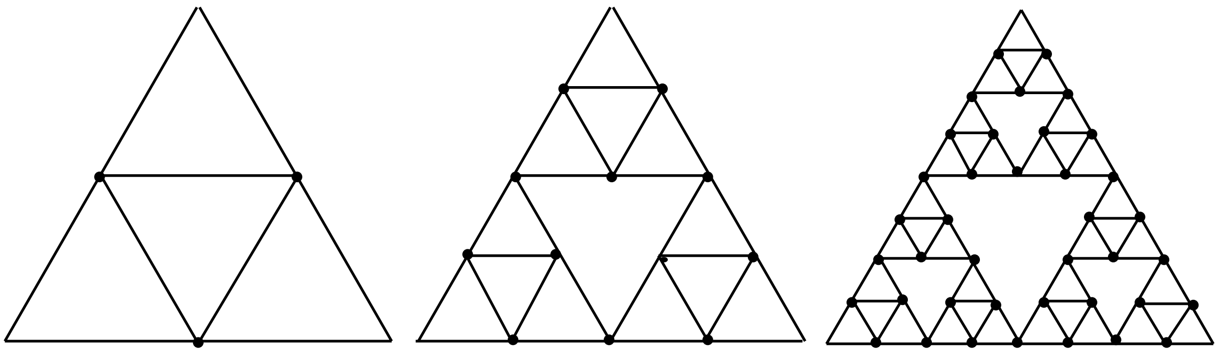
3. After completing the triangle, find the midpoint of each side using the perpendicular bisector technique.

4. Connect the three midpoints **using a straightedge.**

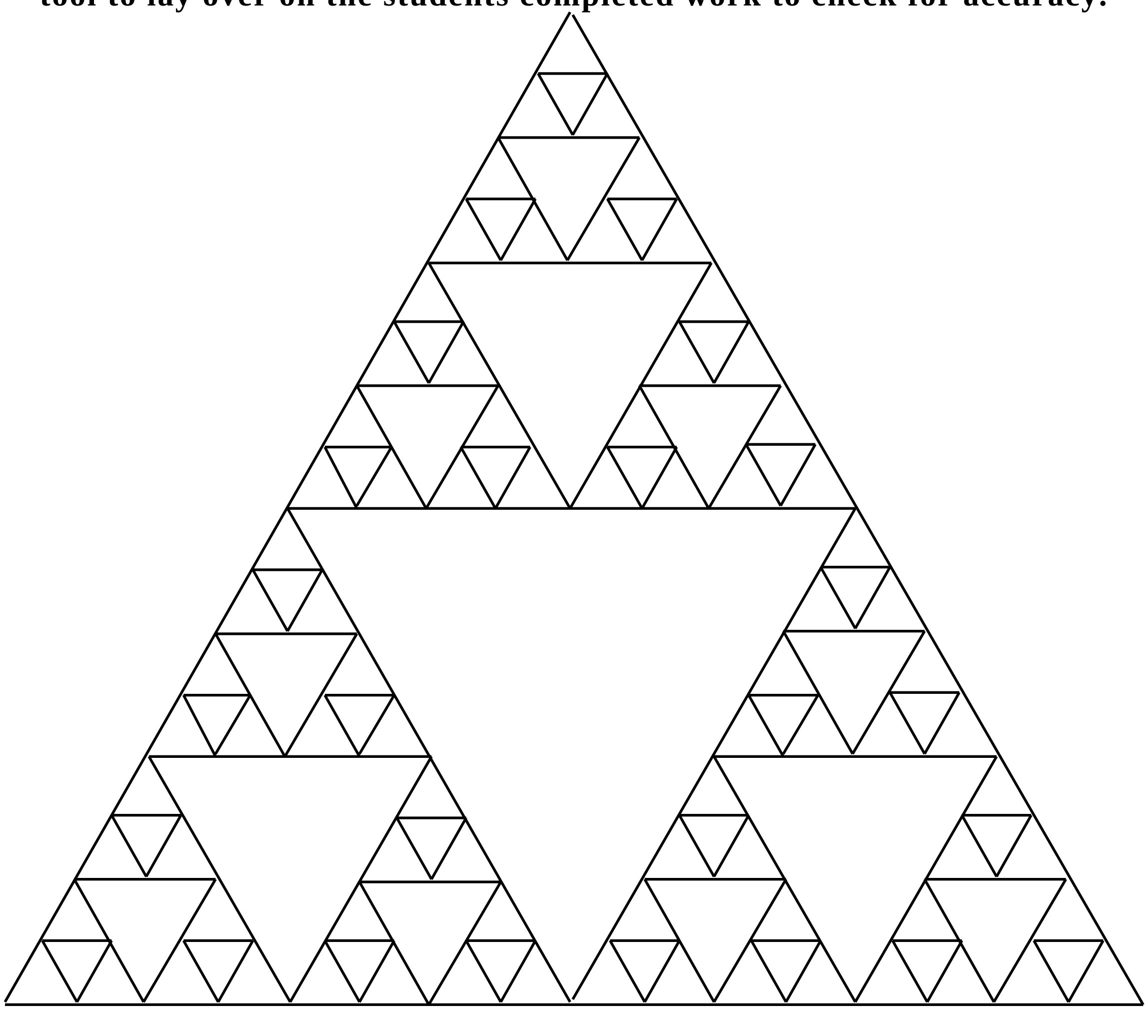
5. Repeat steps 3 & 4 with each remaining upright triangle until the side length of the newly created upside down triangle is less than $^{1}/_{2}$ inch.

Teacher's Notes

Below are some examples of the stages in completing, as well as a completed Sierpinski's Triangle for you to use as a reference in grading.



The triangle below is a full scale replica of the one the students are to create/construct. You may wish to make an overhead to use as a grading tool to lay over on the students completed work to check for accuracy.



The Chaos Game

Materials:

Transparencies
Overhead Projector
Overhead Markers - Red, Blue, Green
Several Dice

Prepare ahead of time for the students:

Several transparencies with all the vertices of a large equilateral triangle marked with clear, large dots, one red, one green, and one blue. All three triangles should be congruent so that the students may overlay their results with other groups to see the final effect.

Playing the game:

1. **Divide the class into groups, one group per prepared transparency. Give each group a transparency and three overhead markers (one each red, green, blue), and a die.**

2. **Instruct the groups to individually select any point on or in the interior of the imagined triangle and to mark it with any of the three pens. This is their "starting point".**

3. **Groups then begin rolling the die and marking their transparency as follows:**

   If the roll is a 1 or 2, select the red pen and red vertex.
   If the roll is a 3 or 4, select the green pen and green vertex.
   If the roll is a 5 or 6, select the blue pen and blue vertex.

4. **Make a new point midway between the "starting point" and the colored vertex selected by the roll. This new dot then becomes the new "starting point".**

5. **Repeat steps 2 - 5.**

After the groups have completed several iterations of steps 2 -5 (at least 15-20), have a member of each group bring their transparency to the overhead and overlay it with the others, being sure to line it up on the vertices. A familiar pattern should emerge.